Key.

Algebra 2 3.1 Polynomial functions

Obj: Evaluate polynomial functions; identify shapes and end behaviors

A **polynomial function** is a monomial or sum or monomials.

Degree: the highest power of all the terms (note if a term has multiple variables sum the powers)

Leading Coefficient: the coefficient with the variable that has the highest power

Standard Form: combine like terms and arrange terms from highest to lowest power

In a polynomial, the exponents are all + whole numbers. (positive)

Common Polynomial names:

Degree	example	Name
0	$y=4$ (x^0)	constant
1	$y=4x$	Linear
2	$y=4x^2+2x+1$	Quadratic
3	$y=4x^3+2x+1$	cubic
4	$y=3x^4+2x+1$	Quartic

Example 1. Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, name, and leading coefficient.

a. $-4x+9+2x^3$ yes

$2x^3-4x+9$

↓

LC: 2

deg: 3

name: cubic

3 terms

b. $2x-3x^4+6-5x^3$ yes

$-3x^4-5x^3+2x+6$

LC: -3

deg: 4

name: Quartic

4 terms

c. $x^5+2x^6-3x^4-8x+4x^3$ yes

$2x^6+x^5-3x^4+4x^3-8x$

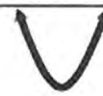
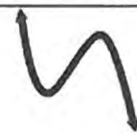
LC: 2

deg: 6

name: 6th degree polynomial

5 terms

END BEHAVIOR: The direction the graph goes as $x \to \infty$ (approaches big positive numbers) or as $x \to -\infty$ (x approaches big negative numbers).

	Odd Degree	**Even Degree**
Positive Leading Coefficient	**Example:** $2x^3$  left y $x \to -\infty$, $f(x) \to$ -∞ right $x \to +\infty$, $f(x) \to$ ∞	**Example:** $2x^2$  left $x \to -\infty$, $f(x) \to$ ∞ right $x \to +\infty$, $f(x) \to$ ∞
Negative Leading Coefficient	**Example:** $-2x^3$  left $x \to -\infty$, $f(x) \to$ ∞ right $x \to +\infty$, $f(x) \to$ -∞	**Example:** $-2x^2$  left $x \to -\infty$, $f(x) \to$ -∞ right $x \to +\infty$, $f(x) \to$ -∞
☆	**If the degree is ODD, then the tails go in <u>opposite</u> directions.**	**If the degree is EVEN, then the tails go in the <u>same</u> direction.** ☆

↑ y→∞
↓ y→-∞
look at left arrow & right arrow

Example 2. Find the End behavior.

look LC & power

a. $p(x) = 2x^3 + 6x - 12$

+ ODD opp.

x→-∞ y→-∞
x→∞ y→∞

b. $q(x) = 5x^4 + 4$

+ even

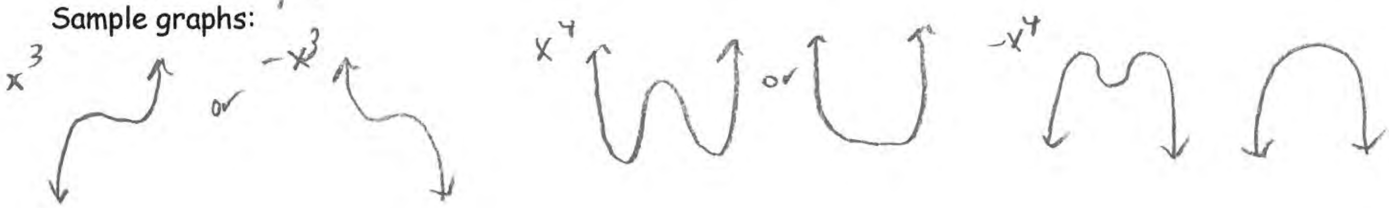
x→-∞ y→∞
x→∞ y→∞

c. $p(x) = -2x^5 + 6x - 12$

neg ODD

x→-∞ y→∞
x→∞ y→-∞

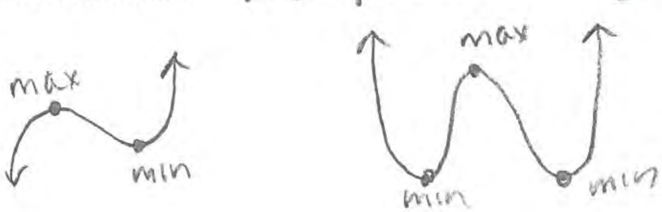
Sample graphs:



Turning points:

Relative maximum: high points on graph where slope change + to -

Relative minimum: low points on graph where slope changes - to +



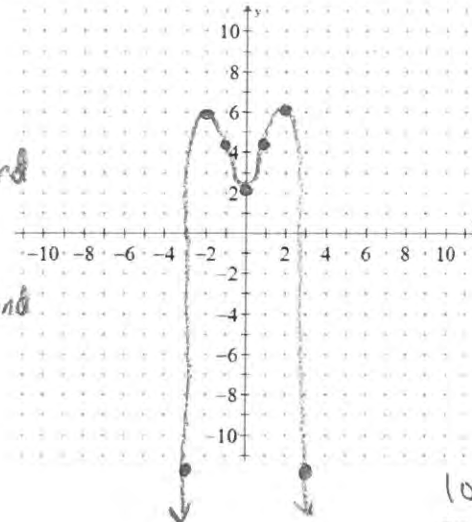
Example 3. Use a table to help identify key features.

Graph. $f(x)=-0.5x^4+3x^2+2$ ✓ ↓ neg even

slope changes

x	f(x)	
-3	-11.5	inc
-2	6	dec
-1	4.5	dec
0	2	inc
1	4.5	inc
2	6	dec
3	-11.5	

rel max around
x = -2
x = 2
rel min around
x = 0



look to check

Estimate the average rate of change from [-2,0]. → slope

$\frac{y_2-y_1}{x_2-x_1}$ (-2,6) (0,2) $\frac{2-6}{0-(-2)} = \frac{-4}{2} = -2$

You try. PUT IT ALL Together. Graph each function using a t table. Describe its general shape.

$P(x) = -x^3 +x^2 +3x-3$

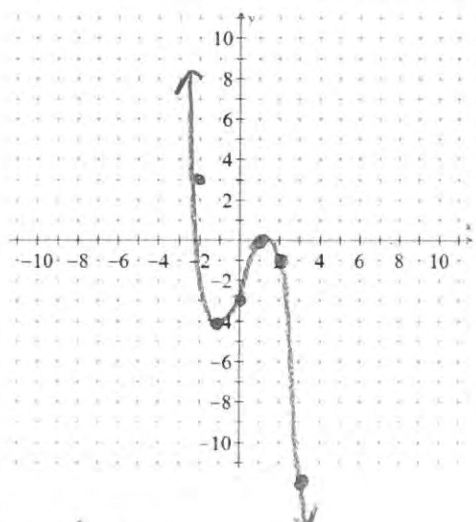
Degree: 3

LC: -1

End behavior: ↑ ↓

$x \to -\infty$ $y \to \infty$
$x \to \infty$ $y \to -\infty$

X	Y
-3	24
-2	3
-1	-4
0	-3
1	0
2	-1
3	-12

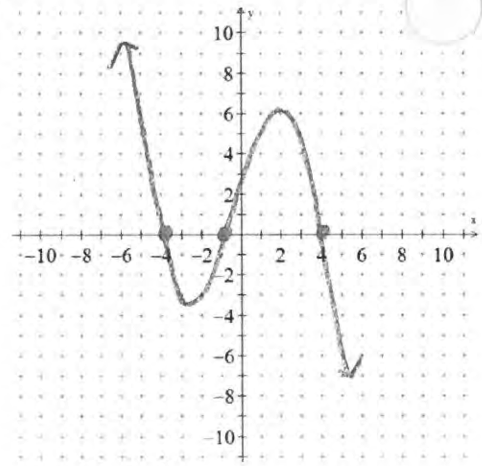


max: around x=1
min around x= -1

Example 4. Sketch the graph from a verbal description.

- *f(x)* is positive on the intervals (-∞,-4) and (-1,4).
  above x axis
- *f(x)* is negative on the intervals (-4,-1) and (4,∞)
  below x axis is so x=
- *f(x)* is decreasing on the intervals (-∞,-2.67) and (2,∞)
  slope neg until x=-2.67
- *f(x)* is increasing on the interval (-2.67,2)
  slope positive

x=-4, x=-1, x=4 are intercepts



Example 5. In science class, Abby mixes a fixed amount of baking soda with different amounts of vinegar in a bottle capped by a balloon. She records the amount of time it takes the gases produced by the reaction to inflate the balloon.

From her data, Abby created a function to model the situation. For *x* quarter-cups of vinegar, it takes $t(x)=-0.12x^3+x^2-3.38x+13.16$ seconds to inflate the balloon.

↑sec.

A. How long would it take to inflate the balloon with 5 quarter-cups of vinegar?

$t(5) = -.12(5)^3+5^2-3.38(5)+13.16$
$= 6.26$ sec.

B. What do the x (cups) and y (time) intercepts mean in the context of this problem? Do they make sense?

x is cups of vinegar. If cups of vinegar is 0, then t would be time the inflate if used no vinegar.

if y=0, the time is 0, so it would be how many cups of vinegar so it would inflate in 0 seconds.